

Objective: Evaluate trig functions of any angle.

Trig Functions of Any Angle

Last week we practiced finding trig functions of angles using the unit circle. This will be similar to the process we used with the unit circle.

The differences:

- The radius is no longer 1 because it isn't the unit circle
- You can use any angle, not just multiples of 30° or 45°
- The coordinate values will be different because the angles are different

Objective: Evaluate trig functions of any angle.

Trig Functions of Any Angle

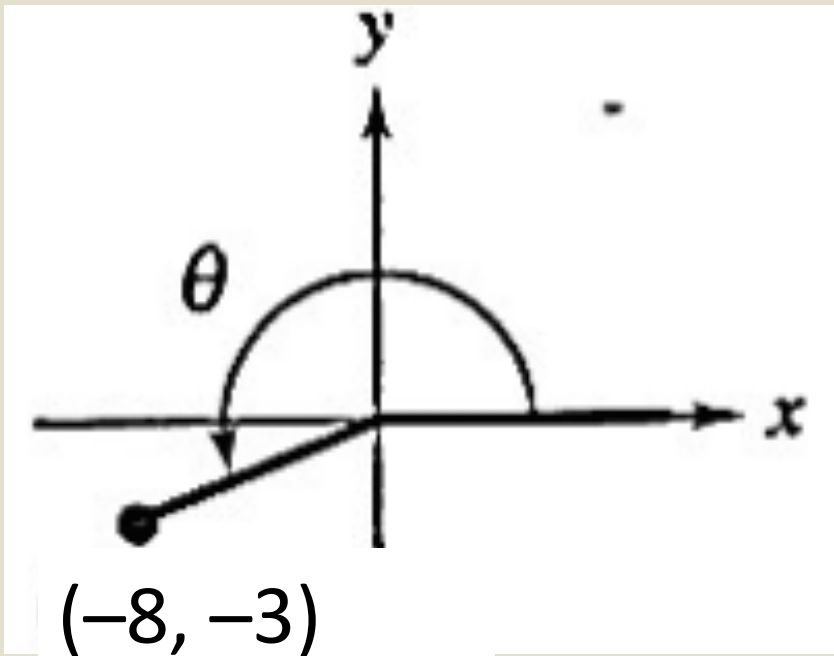
Example 1:

Let $(-8, -3)$ be a point on the terminal side of the angle. Find the exact values of the six trigonometric functions of θ

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Trig Functions of Any Angle

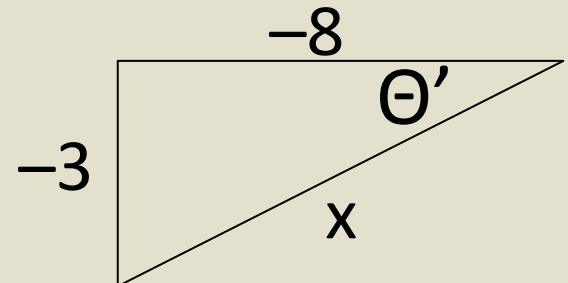
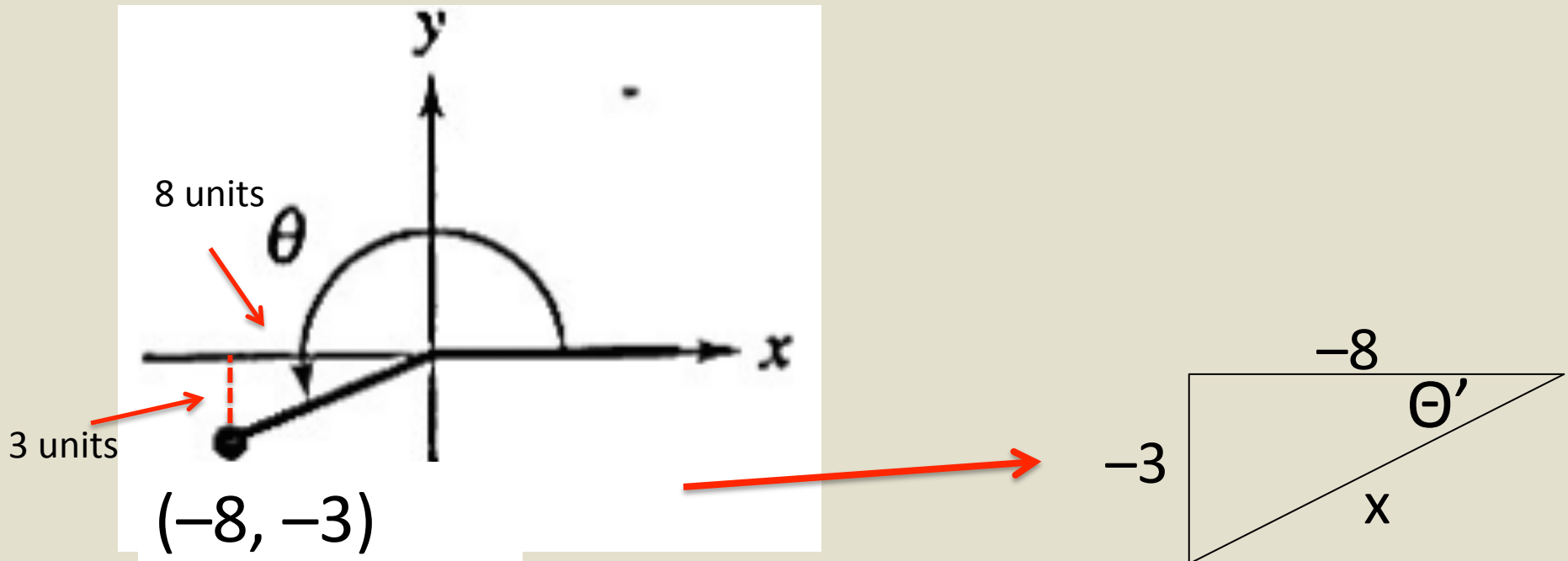
First we need to sketch the angle. The point x and y values tell us the location of the terminal side of the angle.



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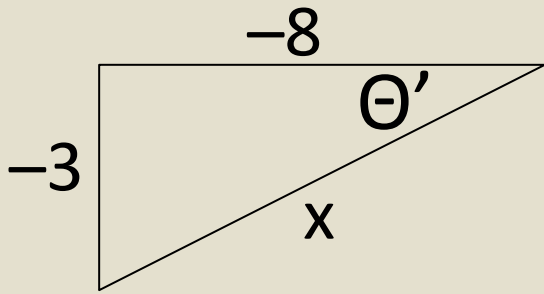
Next we make a right triangle using the x-axis and the reference angle.

This gives us the side lengths for x and y but we need to calculate the hypotenuse.



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Solve for x using Pythagorean Theorem



$$(-3)^2 + (-8)^2 = x^2$$

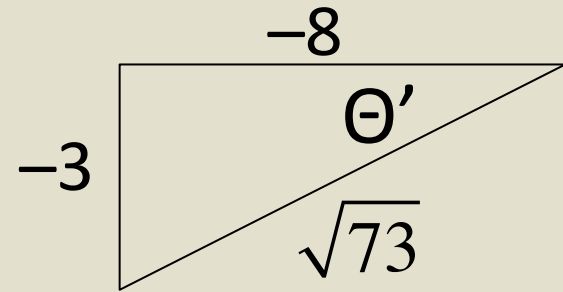
$$9 + 64 = x^2$$

$$73 = x^2$$

$$x = \sqrt{73}$$

Objective: Evaluate trig functions of any angle.

Now we can use the figure to find the exact values of the six trigonometric functions of θ . Be sure to rationalize the denominators when necessary!



$$\sin \theta = -\frac{8\sqrt{73}}{73}$$

$$\cos \theta = -\frac{3\sqrt{73}}{73}$$

$$\tan \theta = \frac{8}{3}$$

$$\csc \theta = -\frac{\sqrt{73}}{8}$$

$$\sec \theta = -\frac{\sqrt{73}}{3}$$

$$\cot \theta = \frac{3}{8}$$

Objective: Evaluate trig functions of any angle.

Example 2:

Given $\cos\theta = \frac{8}{17}$ and $\tan\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).

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Example 2:

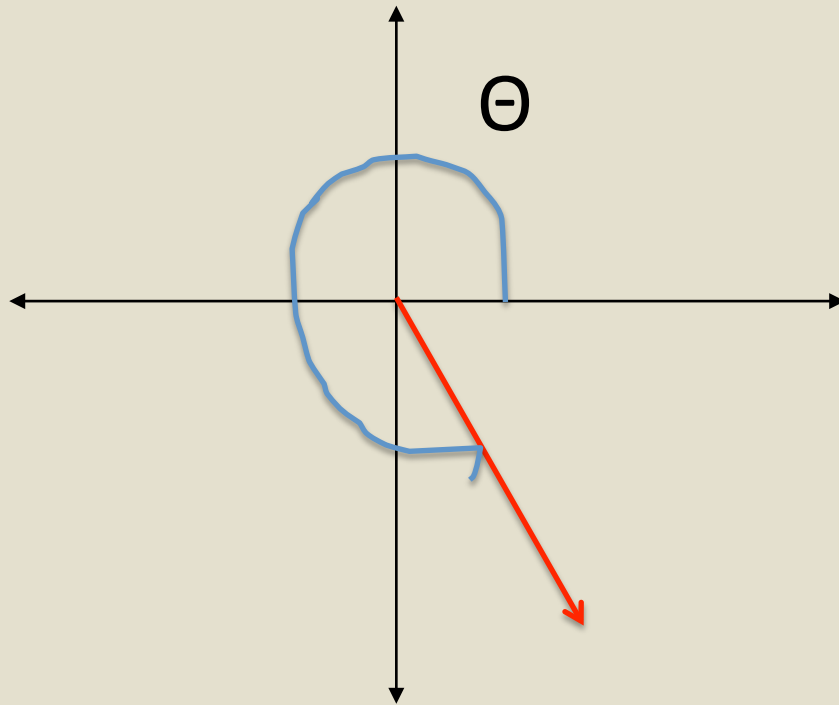
Given $\cos\theta = \frac{8}{17}$ and $\tan\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).

We have to look at the sign of cos and the sign of tan to figure out the quadrant location of the terminal side of the angle. Cos is positive and tan is negative. This means the angle lies in the 4th quadrant. Please review yesterday's assignment (week 4 day 2) if you are not sure about this or reach out during zoom office hours.

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Example 2:

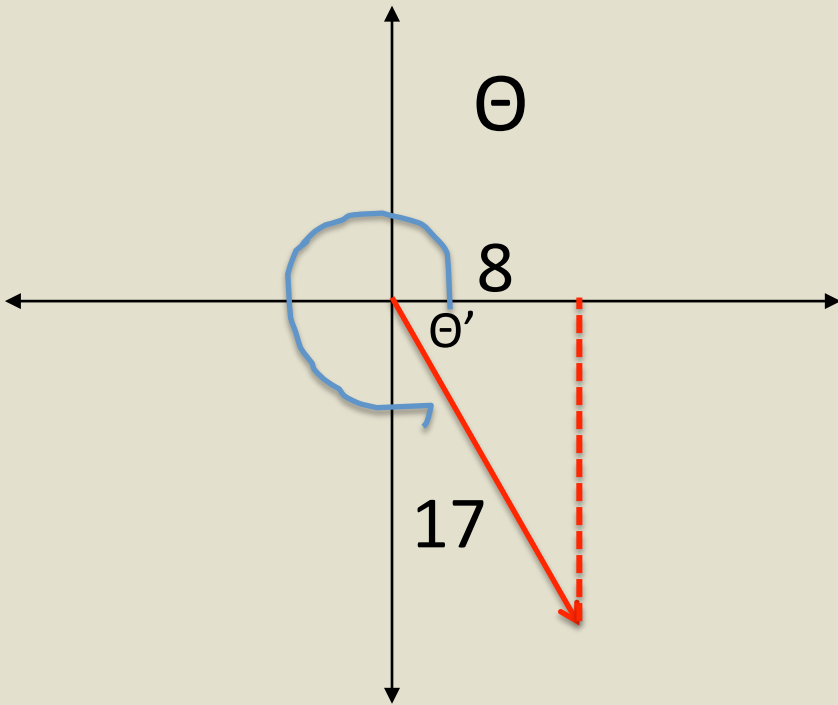
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Given $\cos\theta = \frac{8}{17}$ and $\tan\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).



Make a triangle using the x-axis and the reference angle.

We know \cos is adj/hyp so we can label the adjacent side (x-value) 8 and the hypotenuse 17.

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Given $\cos\theta = \frac{8}{17}$ and $\tan\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).

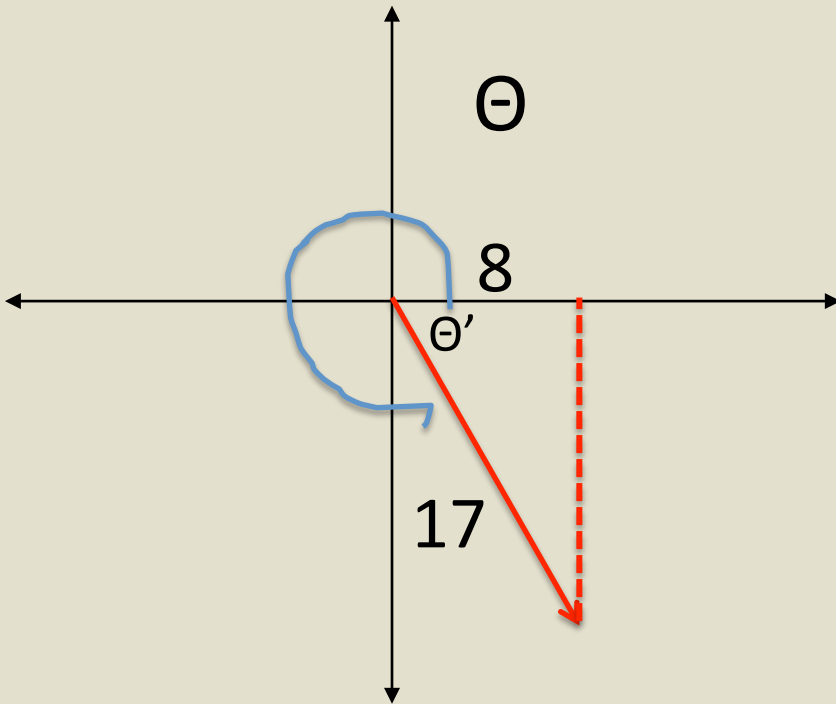
Now use Pythagorean theorem to calculate the 3rd side (the y-value)

$$(8)^2 + (y)^2 = 17^2$$

$$64 + y^2 = 289$$

$$y^2 = 225$$

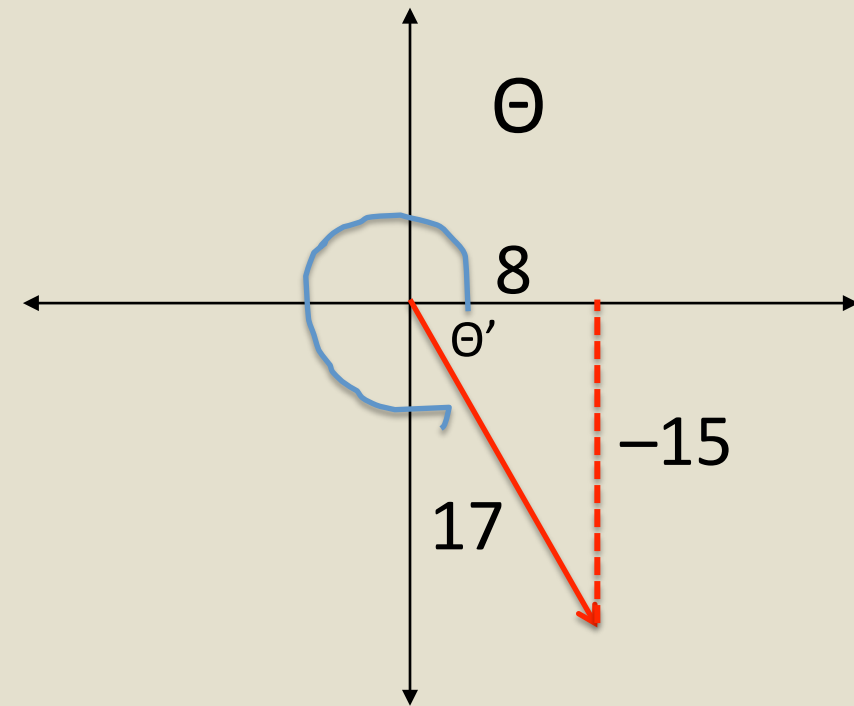
$$y = 15$$



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Example 2:

Given $\cos\theta = \frac{8}{17}$ and $\tan\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).

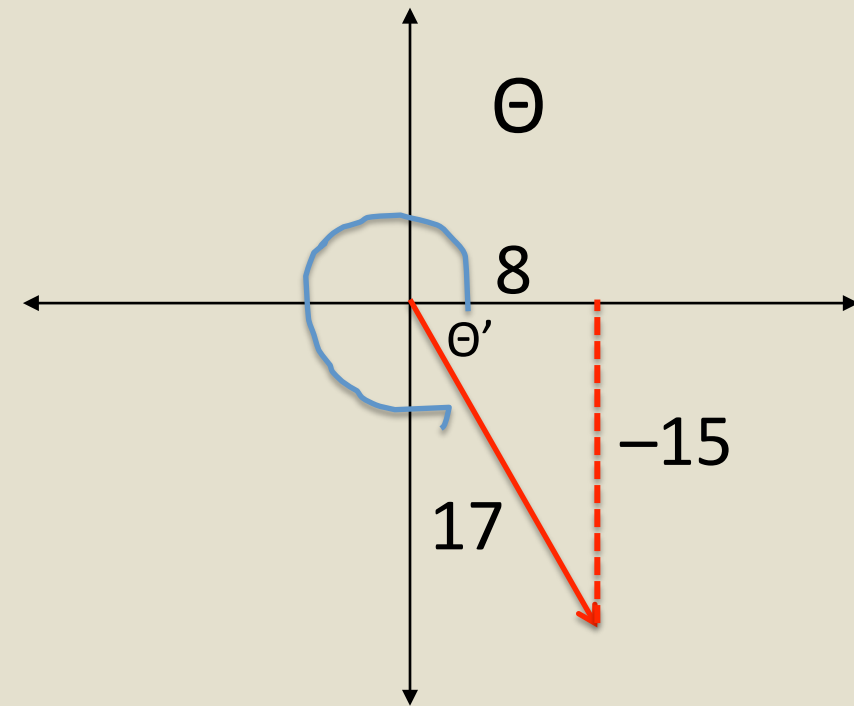


The y value is actually -15 because we have to make it match our angle in quadrant IV.

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Example 2:

Given $\cos\theta = \frac{8}{17}$ and $\tan\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).

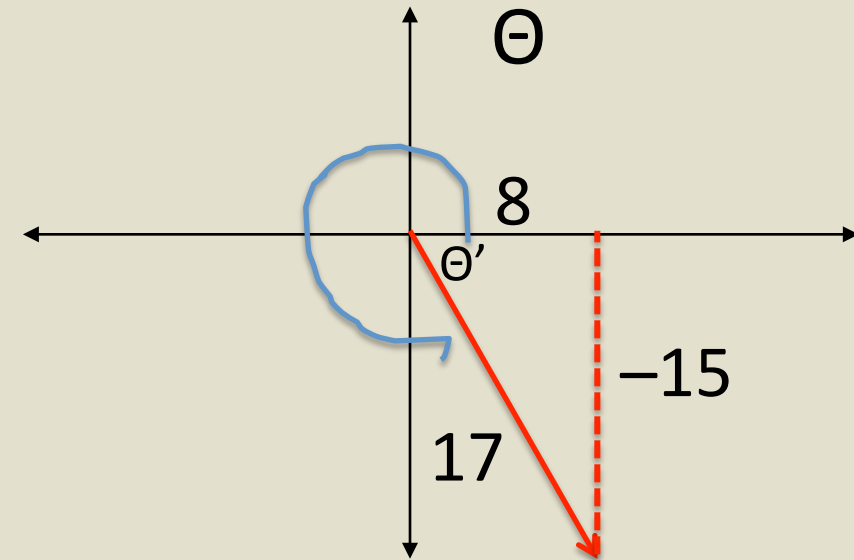


The y value is actually -15 because we have to make it match our angle in quadrant IV.

Objective: Evaluate trig functions of any angle.

Example 2:

Given $\cos\theta = \frac{8}{17}$ and $\tan\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).



Now we can give the 5 trig functions

$$\sin\theta = -\frac{15}{17}$$

$$\tan\theta = -\frac{15}{8}$$

$$\sec\theta = \frac{17}{8}$$

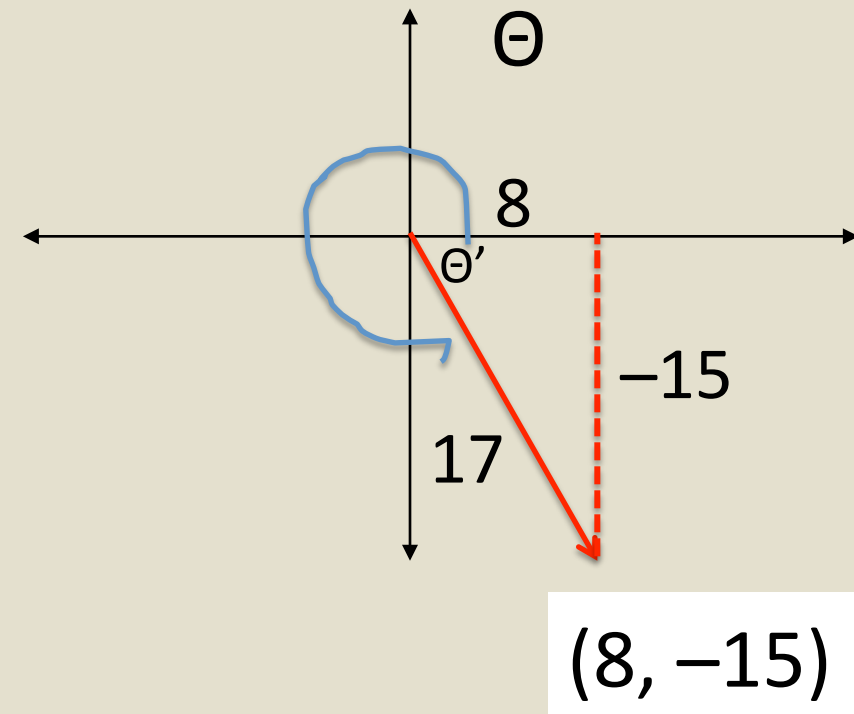
$$\csc\theta = -\frac{17}{15}$$

$$\cot\theta = -\frac{8}{15}$$

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Example 2:

Given $\cos\theta = \frac{8}{17}$ and $\tan\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).



The problem didn't ask, but also note that the point on the terminal side of the angle can now be determined (like the starting point of example 1) because we know the x and y values.

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Example 3:

Now you try: I will solve with explanations on the following slides.

Given $\csc \theta = 4$ and $\cot \theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).

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Example 2:

Given $\csc\theta = 4$ and $\cot\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).

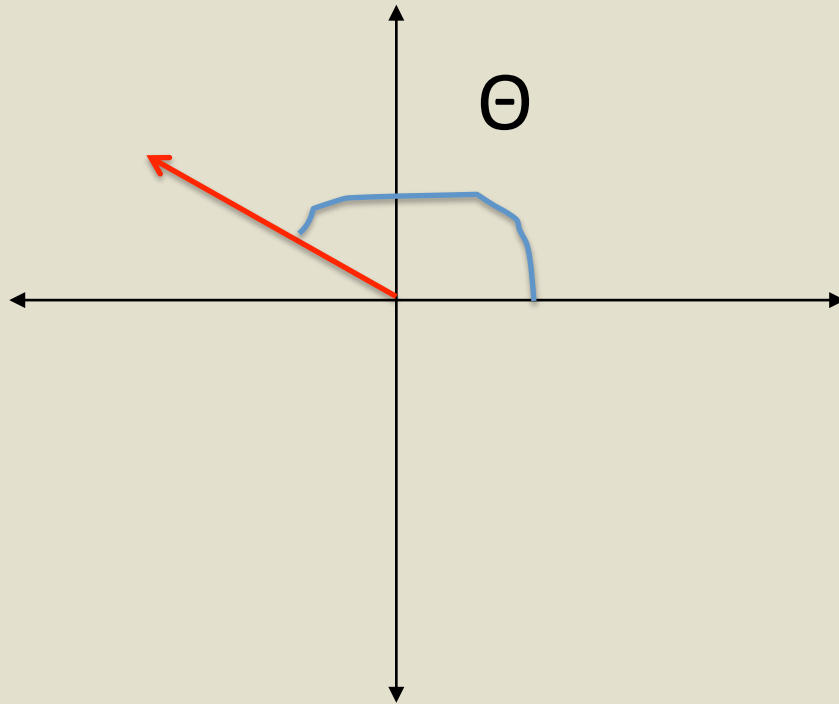
We have to look at the sign of \csc and the sign of \cot to figure out the quadrant location of the terminal side of the angle.

\csc (same sign as \sin) is positive and \cot (same sign as \tan) is negative. This means the angle lies in the 2nd quadrant.

Objective: Evaluate trig functions of any angle.

Example 2:

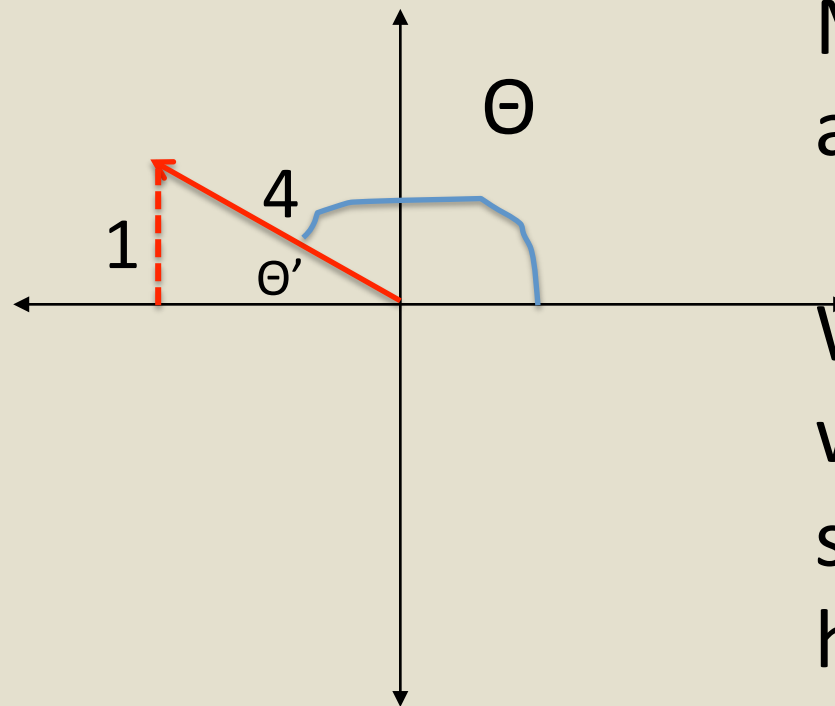
Given $\csc\theta = 4$ and $\cot\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).



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Given $\csc\theta = 4$ and $\cot\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).



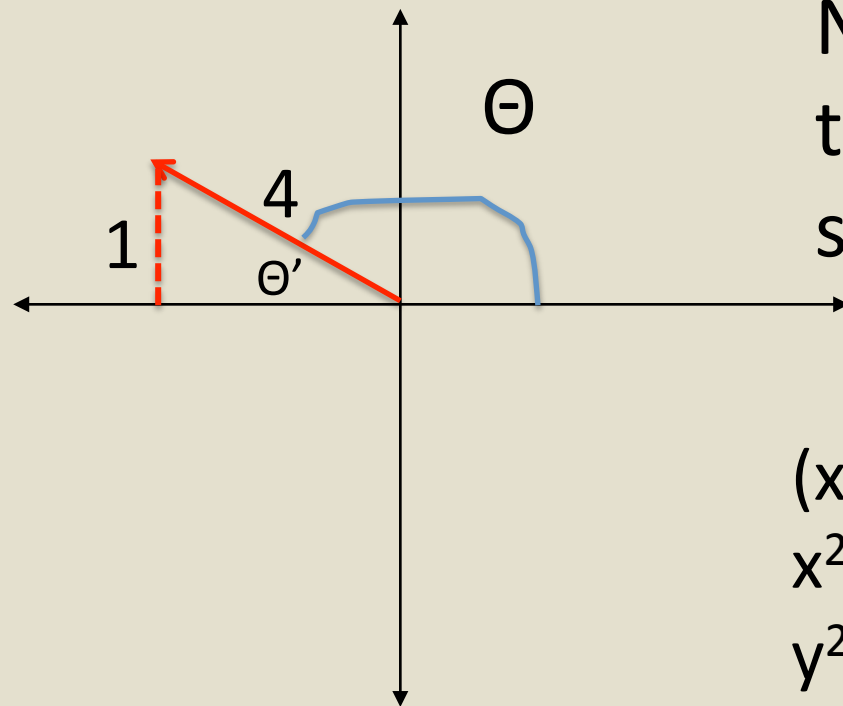
Make a triangle using the x-axis and the reference angle.

We know \csc is hyp/opp so we can label the opposite side (y-value) 1 and the hypotenuse 4.

Objective: Evaluate trig functions of any angle.

Example 2:

Given $\csc\theta = 4$ and $\cot\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).



Now use Pythagorean theorem to calculate the 3rd side (the y-value)

$$(x)^2 + (1)^2 = 4^2$$

$$x^2 + 1 = 16$$

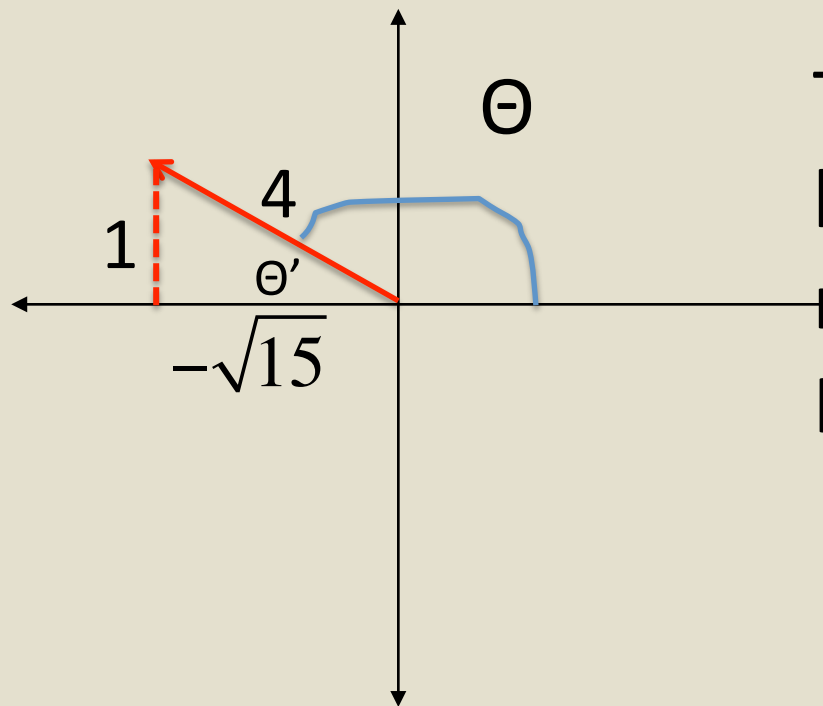
$$y^2 = 15$$

$$y = \sqrt{15}$$

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Example 2:

Given $\csc\theta = 4$ and $\cot\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).

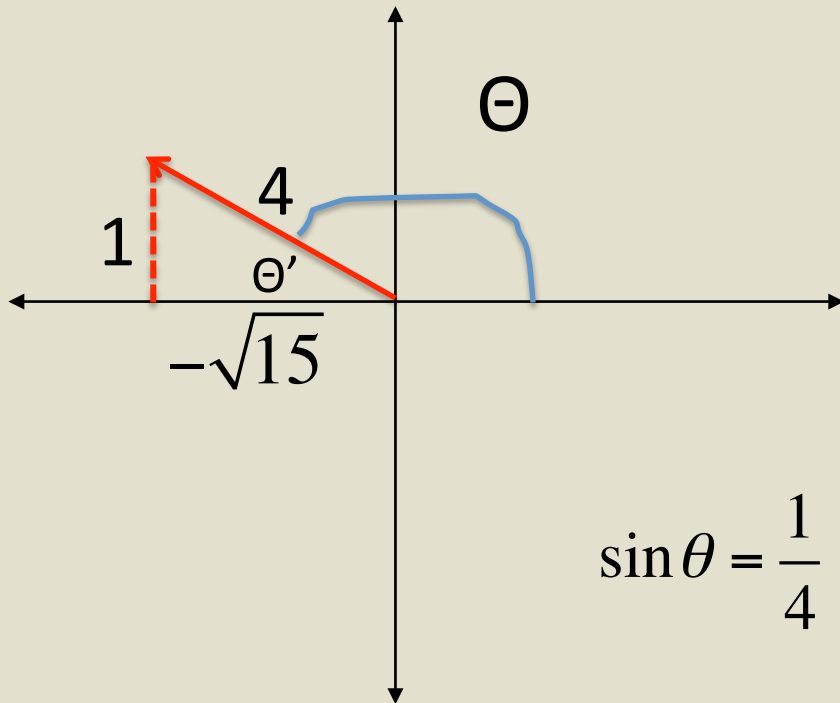


The y value is actually $-\sqrt{15}$ because we have to make it match our angle in quadrant IV.

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Example 2:

Given $\csc\theta = 4$ and $\cot\theta < 0$, sketch the angle and find the 5 remaining trig functions (give exact values).



Now we can give the 5 trig functions

$$\sin\theta = \frac{1}{4}$$

$$\cos\theta = -\frac{\sqrt{15}}{4}$$

$$\tan\theta = -\frac{\sqrt{15}}{15}$$

$$\sec\theta = -\frac{4\sqrt{15}}{15}$$

$$\cot\theta = -\sqrt{15}$$

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Assignment:

Log in to Office 365/Teams

Complete the assignment:

Trig Functions of Any Angle (Week 4, Day 3)

SHOW ALL YOUR WORK!

NO WORK = NO CREDIT